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Effect of Randomness on Outpatient Waiting Times

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The imposition of a guaranteed 13 week wait for first outpatient appointments has led many managers to calculate the required number of first appointments from the average expected number of GP referrals and the average DNA rate for that specialty. This approach can lead to unexpected breaches in the waiting time target. This article explains why this is so and how it can be avoided.

Most people imagine that all NHS outpatient clinics deal with very high numbers of patients per year. This image is reinforced by the fact that the typical outpatient department is used by many different consultant clinics and the flow of patients is a mixture of both first and follow-up. The fact that the size, in terms of first patient attendance, for many of these clinics can be relatively small is lost from sight.

With the assumption of size comes a further assumption of stability, i.e. randomness in GP referrals or DNA rates has little effect on the operation of the clinic. Once again most people forget that even the largest specialties such as T&O and General Surgery are made up from individual consultant clinics. The busiest consultant will only receive around 100 referrals per month, i.e. around 1,200 per annum. While this average varies from month to month due to the existence of monthly profiles for GP referrals (1) this does not effect annual totals.

This relatively moderate size is further diminished by the fact that many consultants run outreach clinics in nearby Community Hospitals and GP Surgeries. Average referrals can be less than 10 to 20 per month. The numbers from these outreach clinics are usually

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mingled with all other clinics in the specialty to give the very much larger numbers reported in NHS statistics.

Having proven the case that the reported numbers may not be a good reflection of the reality of individual NHS clinics we now need to understand how randomness in the GP referrals, DNA rates and other events can have a fundamental influence on the performance of the clinic. In particular, we will examine the ease with which a clinic can meet targets such as a guaranteed 13 week wait for a first appointment.

Randomness

Randomness in the majority of healthcare processes can be described by Poisson statistics (2). Poisson statistics is the branch of statistics used to describe the variation around the average for arrival events such as telephone calls arriving at a switchboard, cars arriving at a motorway exit, GP referrals, emergency admissions, etc. The average arrival rate is determined by measurement while the variation in the actual arrival rate is described by Poisson statistics.

Poisson statistics is also used to describe the dynamic behaviour of queues. Just as queues in a supermarket or bank can form and disperse due to randomness in the number arriving we can also understand why the waiting time for an outpatient appointment may change over time even when adequate resources have been provided to meet the expected average rate of arrival.

Of relevance to outpatient waiting times are two interesting features of Poisson statistics. Firstly, the variation around the average is skewed with a long tail of high numbers counterbalanced by a higher frequency of occurrences less than the average. For example, a consultant receiving an average of 10 referrals per month can receive up to a maximum of 22 referrals in a month but will find that on 58% of occasions they will be receiving 10 or less referrals per month. This skew explains the slight difference between the maximum and minimum numbers given in Table One. This underlying skew in the

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number of GP referrals can also lead to perverse rewards for perceived ‘good’ performance which have arisen due to randomness. For example, with the current focus on outpatient waiting times managers can be ‘rewarded’ or ‘punished’ for changes in waiting time. It is often not appreciated that much of the change in waiting time is due to randomness and is not a ‘fault’ of management.

Provided there are no obvious problems due to insufficient capacity to meet demand then the nature of the skew in Poisson statistics also implies that in any one year more clinics will show an apparent reduction in waiting time than those that show an increase – wrongly interpreted as evidence for good performance. A few clinics will also experience large increases in waiting time, i.e. the tail in a Poisson distribution leading to high numbers – also wrongly interpreted as poor performance.

Another interesting fact about Poisson statistics is that the standard deviation (a measure of the variation around the average) is equal to the square root of the average rate of arrival. Hence if we are expecting an average of 100 GP referrals per month then the standard deviation will be 10. We should not be surprised if we actually receive anywhere between 70 and 130 referrals in any particular month since the full range in referrals is approximately described by the average \pm 3-times the standard deviation.

Remember that 100 per month is the volume of referrals expected for the largest consultant clinic. For a consultant clinic expecting only 25 referrals per month then the range in referrals will be between 10 and 40. The size of the clinic therefore has a dramatic effect on the variation around the average and hence on the fluctuation in waiting time experienced from month to month.

It should be obvious from the above discussion that simple planning based on the average will invite disaster, simply because the level of randomness is too high even for the largest consultant clinic. These considerations suggest that we should explore the possible roles that randomness could play in the management decisions required for outpatient clinics.

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Annual volume of GP referrals

Having determined that the largest consultant clinic receives fewer than 200 referrals per month it is possible to look at the impact of randomness on the annual total. This has been done in Table One where the maximum and minimum number of referrals has been shown as a function of clinic size. A word of caution, the variation associated with annual numbers is not the sum of monthly variation. This is because the standard deviation associated with any average (e.g. the annual total) is the square root of itself.

As can be seen from Table One the largest consultant clinic will experience up to 6% variation (i.e. half the range) around the expected average number of GP referrals for that year. If this clinic is resourced to handle the average expected number of GP referrals (e.g. 2,400 first appointments per year) then it is possible that up to 149 extra patients could be waiting for a first appointment by year-end. This is the worst possible outcome and would act to increase the waiting time by 3.2 weeks.

Hence if the waiting time at the start of the year were 13 weeks it would rise to 16.2 weeks by year-end. This is not the result of poor planning or staff not working hard enough but simply an outcome of randomness. Obviously randomness could lead to a range of outcomes between a 3.2 week increase and a 3.2 week reduction in the waiting time, however, it is the avoidance of an increase in waiting time which is the important consideration when a maximum waiting time has been imposed.

Also evident from Table One is the fact that any clinic anticipating fewer than 170 GP referrals per annum can go from a zero waiting time to over 13 weeks waiting time – simply due to randomness in GP referrals.

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Table One: Variation in GP referrals per annum due to randomness

| Average Expected GP referrals | Maximum possible | Minimum possible | Range in GP referrals (Max - Min) | | Highest possible change in waiting time over one year | |
|-------------------------------|------------------|------------------|-----------------------------------|--------------|---|-----------------|
| | | | Number | % of average | Maximum (weeks) | Minimum (weeks) |
| 2400 | 2549 | 2254 | 295 | 12% | 3.23 | -3.16 |
| 2000 | 2137 | 1866 | 271 | 14% | 3.56 | -3.48 |
| 1000 | 1097 | 906 | 191 | 19% | 5.04 | -4.89 |
| 900 | 992 | 810 | 182 | 20% | 5.32 | -5.20 |
| 800 | 887 | 716 | 171 | 21% | 5.66 | -5.46 |
| 700 | 782 | 621 | 161 | 23% | 6.09 | -5.87 |
| 600 | 676 | 527 | 149 | 25% | 6.59 | -6.33 |
| 500 | 570 | 433 | 137 | 27% | 7.28 | -6.97 |
| 400 | 462 | 340 | 122 | 31% | 8.06 | -7.80 |
| 300 | 354 | 249 | 105 | 35% | 9.36 | -8.84 |
| 200 | 245 | 157 | 88 | 44% | 11.70 | -11.18 |
| 170 | 212 | 131 | 81 | 48% | 12.85 | -11.93 |
| 150 | 189 | 114 | 75 | 50% | 13.52 | -12.48 |
| 130 | 167 | 96 | 71 | 55% | 14.80 | -13.60 |
| 100 | 132 | 70 | 62 | 62% | 16.64 | -15.60 |
| 90 | 120 | 62 | 58 | 64% | 17.33 | -16.18 |
| 80 | 109 | 54 | 55 | 69% | 18.85 | -16.90 |
| 70 | 97 | 46 | 51 | 73% | 20.06 | -17.83 |
| 60 | 85 | 37 | 48 | 80% | 21.67 | -19.93 |
| 50 | 73 | 29 | 44 | 88% | 23.92 | -21.84 |
| 40 | 61 | 21 | 40 | 100% | 27.30 | -24.70 |
| 30 | 48 | 14 | 34 | 113% | 31.20 | -27.73 |
| 20 | 35 | 7 | 28 | 140% | 39.00 | -33.80 |
| 10 | 21 | 1 | 20 | 200% | 57.20 | -46.80 |

These observations have important implications to the way in which clinics of different size are run. It is randomness in GP referrals that dictates the appropriate clinic structure.

Pooling referrals

The larger specialties are in the enviable position of being able to pool their referrals. The GP's are advised to send referrals addressed to 'Dear Consultant' and in this way the referral can be allocated to the Consultant with the shortest wait. In effect this allows the specialty to benefit from economies of scale and the referral volume is thus the combined volume of all consultants. This pooling can only occur when any consultant can see the referral. When there are sub-specialties within a specialty the referral volume is for the sub-specialty.

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Hence, for a large specialty with five Consultants and 5,000 referrals per annum the variation is reduced to ± 212 (3-times the square root of 5,000) which is equivalent to a 4% variation. A 4% variation is still of considerable size and indicates that all NHS planners should be aware of the central role of randomness in the performance of outpatient clinics.

Small clinics (fewer than 200 referrals per annum)

Small clinics are effectively unable to operate a fixed weekly appointment system. They must be structured in either of two ways. Firstly they could be structured in such a way that the number of appointment slots offered next month would match the number of referrals received this month. Using this strategy demand and supply are always matched even in the face of high random fluctuation in referrals. The alternative structure is to operate at the expected average but to run additional clinics whenever demand is higher than expected due to randomness.

Intermediate sized clinics (between 200 and 800 referrals per annum)

These clinics will have to use a mixture of strategies to manage the waiting time. The first strategy is to offer more clinic slots than the expected average number of GP referrals. A good starting point would be to increase the average number by 1.5-times the standard deviation (equivalent to one-quarter of the range). The figure of one-quarter of the range actually covers 91% of all possible outcomes, i.e. there would only be more GP referrals than this figure in 1 out of 10 years. For example, a clinic expecting 800 GP referrals would attempt to accommodate 843 new patients per annum while a clinic expecting 300 GP referrals would attempt to accommodate 326 new patients per annum.

On the 9% of occasions when demand may be higher than expected due to randomness then additional clinic slots would have to be provided on a temporary basis (depending on the prevailing waiting time).

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Larger clinics (over 800 GP referrals per annum)

For the larger clinics it is usually sufficient to offer a number of clinic slots equal to the expected average referrals (see below for adjustment for the effect of DNA). However, to avoid any year-end breach of the 13 week target, the waiting time at the beginning of the year should be 'x' weeks lower than the 13 week target. The value of 'x' coming from the 6th column in Table One.

For example, a clinic expecting 700 GP referrals would need to have a wait of 7 weeks (13 – 6) at the start of the year to avoid any breach of the waiting time target due to randomness in GP referrals. A clinic expecting only 400 GP referrals would need to start the year at a waiting time less than 5 weeks. If this condition cannot be satisfied then additional clinic slots will have to be offered throughout the year to bridge any gap.

Effect of DNAs

DNAs complicate the situation considerably. Similar to GP referrals they are also subject to Poisson randomness and so Table One can be used to predict the range in DNAs expected across a whole year. Once again a prudent approach needs to be taken and the maximum possible DNAs must then be added to the anticipated number of first appointment slots in order to guarantee no increase in the waiting time.

For example, assuming a 10% DNA rate (equivalent to 11.1% of appointment slots lost due to non-attendance) the largest clinic from Table One would be expecting an average of 267 DNAs for the year. Using 300 as the closest figure in Table One gives us 54 more DNAs as the maximum possible DNAs in the year. We can use half this figure (the same as one quarter of the range) to cover 91% of all outcomes, thus adding 27 to 267 gives 294 additional appointment slots to be provided to avoid any increase in the waiting time due to DNAs and randomness in the DNA rate.

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Our total provision of appointment slots for the year then becomes 2,400 (average GP referrals) + 75 (randomness in GP referrals) + 267 (average DNAs) + 27 (randomness in DNAs). The largest consultant clinic would therefore, in order to guarantee no increase in waiting time, have to provide 2769 first appointment slots in order to treat an average demand of 2,400 GP referrals. This represents 15% more appointment slots than the demand it is intended to satisfy!

The salutary lesson in this is to reduce the DNA rate to the minimum possible value. Methods for targeting specific clinics and locations based on patient age, sex and socio-economic variables are available (3).

How many GP referrals next year?

In most specialties the volume of GP referrals received from year to year increases in a linear manner (1). On occasions the linear trend will show a step upward due to new technology or the arrival of an additional consultant. Forecasting the average expected for next year is then relatively straightforward. A simple linear regression (or even a line of best fit drawn by eye) can be performed using the past five or more years data on GP referrals. Where there is a step upward the magnitude of the step can simply be added to the years previous to the step and the regression performed on this adjusted data.

The resulting average is itself uncertain due to the randomness in the number of referrals in any year and strictly speaking an additional allowance should be made to account for this uncertainty in the calculated average. However, the additional allowance provided by randomness around the average and randomness in DNAs should be sufficient.

Effect of Other Adverse Events

GP referrals can also show additional variation due to other adverse events such as epidemics, weather, etc. These events usually act to increase referrals above the expected average for several months, i.e. they act to increase demand. Other random events such as

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Consultant unavailability (illness, jury service, study leave, etc), equipment breakdown, etc act to restrict the supply of available appointments and would also act to increase the waiting time. To avoid any breach of waiting time due to adverse events implies that the waiting time at the start of the year needs to be reduced by an extra margin over-and-above that required for randomness in GP referrals and DNA rates.

Meeting a year-end target

Given that the maximum waiting time target is 13 weeks it then follows that no GP referral received after the 1st January can count toward the year-end number of long waiting patients. This is because the 30th March (last clinic day of the financial year) is less than 13 weeks away from the 1st of January.

The number of GP referrals received in December can therefore be a critical factor in determining ultimate year end performance. Taking the largest possible consultant clinic as having 2,400 referrals per annum gives roughly 200 referrals in December. Using Table One we can see that the actual number of referrals received can be anywhere between 157 and 245. If clinic capacity is 200 per month then the number of patients waiting longer than 13 weeks has the potential to swing between –43 to +45 simply due to randomness in GP referrals received in December. To avoid the worst case of a random increase of +45 implies offering 45 extra clinic slots in December, i.e. a 22% increase in usual clinic capacity for the month.

This is the best possible case as the situation becomes more unpredictable as clinic size reduces. Your ability to achieve a year end target is therefore dependant, to a large degree, on chance.

Conclusions

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Randomness in GP referrals and DNAs is shown to dictate the strategies required to achieve a set waiting time target. A table of values has been provided to allow calculation of outpatient volume required in order to guarantee a waiting time.

In effect, to categorically avoid breaching a 13 week target most large clinics will need to operate at an average wait of lower than 10 weeks while smaller clinics must have the flexibility to offer additional appointments as required.

The ability of a clinic to meet a year-end 13 week target can be dependent on the relatively high randomness associated with GP referrals received during December.

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